Fluency, Accuracy, and Gender Predict Developmental Trajectories of Arithmetic Strategies

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The purpose of this study was to determine whether there are different growth trajectories of arithmetic strategies and whether these trajectories result in different achievement outcomes. Longitudinal data were collected on 240 students who began the study as 2nd graders. In the 1st year of the study, the 2nd-grade students were assessed on fluency and accuracy on simple arithmetic problems. During the fall of the 2nd, 3rd, and 4th grades, they were assessed on strategies for complex (multiple-digit) arithmetic. They were assessed on mathematics competency at the end of 4th grade. Growth mixture modeling was used to assess developmental trajectories in arithmetic strategies; the roles of fluency, accuracy, and gender in the development of latent class; and the impact of latent class on 4th-grade mathematics competency. The data indicated 2 latent classes of growth trajectories for correct cognitive strategy use and for attempted manipulative strategy use. Three latent classes were needed to explain the development of attempted cognitive strategy use. Fluency appeared to have the most significant impact on the growth rate, whereas accuracy and gender tended to influence the initial level of performance. Not all children transitioned away from manipulatives easily. A small latent class of children slightly increased their use of manipulatives over the course of the study, with the girls in this group being less likely than boys to abandon manipulatives. Finally, what appears to be the normal developmental trajectory for strategies was not found to serve many children well in regard to later mathematics achievement.

Keywords: mathematics achievement, fluency, strategies, gender, arithmetic

During early elementary school, children’s arithmetic strategies transition from the use of manipulatives to the use of cognitive strategies to the use of retrieval as means of solving arithmetic problems (Carpenter & Moser, 1984). It is not clear what factors influence these transitions or how the timing of the transitions predicts later achievement. In particular, we do not know whether all children experience these transitions in the same way or whether there are different developmental trajectories. Furthermore, most of the work done to date on the development of arithmetic strategies has focused on younger children solving simple, single-digit arithmetic problems (e.g., Geary, 1987; Geary, Salihouse, Chen, & Fan, 1996; Kerkman & Siegler, 1997; Widaman, Little, Geary, & Cormier, 1992). We know less about the development of arithmetic strategies for complex, multidigit problems that are acquired during the second grade as children are introduced to more complex problems. In the 3-year longitudinal study described in this article, we examined whether there are different trajectories in the development of arithmetic strategy use for complex problems and whether differences in developmental trajectories influence performance on a fourth-grade mathematics competency test. As such, we explore a period of time between the second and fourth grades when strategies for complex arithmetic emerge and evolve and the impact of that development on mathematics competency. The study also reports on the impact of several variables, including gender, fluency, and accuracy, on the trajectory of arithmetic strategies and the impact of that trajectory on mathematics competency.

For the purposes of this study, arithmetic strategies are defined broadly as any procedure used to solve an arithmetic problem that can result in a correct answer. Children may count on fingers, try to retrieve the answer from memory, mentally calculate the answer to arithmetic problems, or use the algorithms taught in classrooms. This definition is more in line with Ashcraft (1990), who defined strategies as a broad range of procedures. It is less in line with the definition of strategy by Bisanz and LeFevre (1990), who preferred a more constrained use of the term so as to discriminate strategies from other cognitive procedures. We use this broader definition because we have found that children often prefer certain ways of solving problems, some of which are typically categorized by researchers as strategies (e.g., counting on fingers) and some of which are not typically categorized as strategies (e.g., retrieval). Given this, procedures such as retrieval from memory are not viewed as automatic responses in which a child retrieves the correct answer from memory but as a choice of a strategy.
Variability of Arithmetic Strategies

There is considerable evidence that children and adults use a variety of arithmetic strategies at any given time and these strategies change over time. Siegler (1996) and Svenson and Sjöberg (1983) proposed that children’s mathematics strategy use develops in a wavelike manner, with some strategies being used more often than others at particular points in time. As new strategies emerge, older strategies are used less often and are sometimes abandoned completely. Research by Ilg and Ames (1951) and Svenson and Sjöberg (1983) supports this view of strategy development for simple arithmetic problems, with young children tending to have preferred strategies that dominate their problem solving and using a variety of strategies that change over time. Variability in strategy use exists even for adults who apply a variety of strategies, ranging from counting on fingers to retrieval, despite having a highly developed number sense and much more experience than children with arithmetic (Groen & Parkman, 1972; Lemaire & Arnaud, 2008). Other research by Siegler (1988) hinted at possible heterogeneity in the development of arithmetic strategies: Some fourth-grade children (perfectionists) were found to use retrieval less often than children with comparable levels of mathematics competence did. Thus, we know there is variability in strategy use, but we do not know whether this variability results in different developmental trajectories of arithmetic strategy use.

There is increasing evidence for the heterogeneity in developmental trajectories in mathematics. For example, Jordan, Kaplan, Oláh, and Locuniak (2006) found three developmental trajectories for number sense in kindergarten children. Aunola, Leskinen, Lerkkanen, and Nurmi (2004) found that high achievers in mathematics had different developmental trajectories than low achievers did as a function of initial counting ability. Work on children with mathematics difficulties (Geary, Hamson, & Hoard, 2000; Gersten, Jordan, & Flojo, 2005) indicates several developmental trajectories, with some children who show poor initial performance catching up to their classmates and other children continuing to show a developmental delay over a period of years. The results of these studies suggest that a single snapshot of performance cannot predict later outcomes and that longitudinal work is needed to understand how arithmetic strategies develop.

We know little about the development of complex arithmetic strategies. Much of the research on complex arithmetic has involved adult populations (e.g., Frensch & Geary, 1993). The work with younger children indicates that although elementary school children use a range of strategies to solve the multidigit arithmetic problems, they tend to shift from counting procedures to retrieval (Widaman et al., 1992). The counting procedures include manipulative strategies (e.g., finger counting) and mental counting; regrouping of hundreds, tens, and ones (decomposition); and the use of the columnar arithmetic (standard algorithm; Fuson, Stigler, & Bartsch, 1988; Geary, 1994; Geary, Hoard, Byrd-Craven, & DeSoto, 2004). Some of these strategies, such as counting on fingers, were developed for use with simple arithmetic and extended to complex arithmetic problems. Other strategies, including the standard algorithm and decomposition, are specific to complex arithmetic and reflect the increased demands of complex arithmetic, including the need to deal with place value during computation. No studies have examined how these strategies develop and change longitudinally in the general elementary school population.

This type of research would provide insight into how variability in strategy development predicts later mathematics outcomes.

Roles of Fluency, Gender, and Accuracy

We know that children acquire new strategies over time, but we know little about the factors that influence the development of arithmetic strategies. Changes in conceptual knowledge about number are thought to drive this transition (Steffe, Cobb, & von Glasersfeld, 1988). Working memory also appears to predict the shift to cognitive strategies and retrieval (Geary et al., 2004). There is also evidence that strategy use is influenced by children’s fluency, accuracy, and gender. Furthermore, gender and fluency are correlated so it would be beneficial to compare their relative contributions to arithmetic strategy use and mathematics competency when both are controlled.

Gender differences have been documented in early mathematics strategy use and have been found to predict developmental trajectories of mathematics achievement. Research by Carr and Jessup (1997) indicated that gender differences in mathematics strategy use emerge during the first grade, with girls tending to use manipulatives and boys preferring retrieval. Other research by Fennema, Carpenter, Jacobs, Franke, and Levi (1998) indicates that these differences continue into the third grade, with girls tending to use manipulatives and boys tending to use cognitive strategies and retrieval. In examining mathematics achievement, Aunola et al. (2004) found evidence that girls’ and boys’ mathematics achievements follow different developmental trajectories, with being male predicting the slope for high-performing students but not for low-performing students. Other research by Leahey and Guo (2001) also indicated that the developmental trajectory for boys’ mathematics achievement was accelerated in contrast to that of girls. These studies support a hypothesis of different developmental trajectories for boys’ and girls’ strategy use and mathematics achievement. It is not clear, however, whether gender determines these developmental trajectories or whether other factors linked to gender differences (e.g., fluency) are responsible.

Computational fluency and the accuracy of students’ answers also influence strategy development and mathematics competency. Poor fluency in retrieval of answers to simple arithmetic problems slows the transition from strategies involving manipulatives to cognitive strategy use and retrieval (Jordan, Hanich, & Kaplan, 2003). Poor-performing students in mathematics typically show poor accuracy in the retrieval of answers to single-digit problems from memory (Goldman, Pellegrino, & Mertz, 1988) and are slower in their arithmetic problem solving (Bull & Johnston, 1997; Royer, Tronsky, Chan, Jackson, & Marchant, 1999). Fluency and accuracy are thought to support problem solving by allowing students to allocate working memory resources to representing and solving problems and through speeded retrieval from memory (Ashcraft, Donley, Halas, & Vakali, 1992; Geary, 1994). Given this, it is likely that children’s fluency, or the speed with which they correctly solve single-digit arithmetic problems, and accuracy, defined as the percentage of those problems that are correctly solved, influence the development of arithmetic strategies and mathematics competency.

Gender differences exist in both fluency and strategy use, so it is not clear whether it is gender or differences in fluency that
produce differences in strategy use. Royer et al. (1999) have proposed that gender differences in mathematics achievement and the tendency of boys to be among the highest scorers on mathematics tests is a function of boys’ higher fluency in solving mathematics problems. Boys are more likely to use retrieval and are more accurate in their retrieval of answers to mathematics problems (Carr & Davis, 2001). By examining the roles of gender, fluency, and accuracy in the development of arithmetic strategies, we were able to better understand the relative contributions of these variables to the development of arithmetic strategies and mathematics achievement.

Current Study

Strategies for complex arithmetic problems have been categorized as verbal counting, regrouping or decomposition, and columnar retrieval or the standard algorithm (Geary, 1994). For the purposes of this study, we used these categories but included additional categories. Over the 3-year period of the study, children used a variety of strategies not included in Geary’s categories, including counting all using manipulatives, counting on either mentally or with the use of manipulatives, counting back for subtraction either mentally or with the use of manipulatives, and the standard algorithm using manipulatives. In contrast to prior research, we found that a substantial number of children used manipulatives of some kind (fingers, hatch marks, or counters) to compute answers.

We collapsed across all strategies that involved a physical representation of number, including the use of fingers, counters, and notations on paper (e.g., hatch marks), into a manipulative strategy category. All strategies that involved mentally representing number (mental counting, decomposition, and the standard algorithm using retrieval or mental counting) and retrieval were included in a second, cognitive strategy category. This categorization was supported by data indicating that, by the fourth grade, elementary school children’s arithmetic strategies form two distinct factors, one comprising manipulation-based strategies and the other comprising cognitive strategies and retrieval (Biddlecomb & Carr, 2011). Furthermore, it was believed that the transition from the physical representation of number to a cognitive representation of number represents a major conceptual shift in mathematical thinking as well as a substantial decrease in working memory load. Specifically, this transition is the result of children’s maturing number sense, including a rich, abstract representation of numbers, their relationships to each other, and related operations (Baroody, Brach, & Tai, 2006; Steffe, Cobb, & von Glasersfeld, 1988). The failure to make the transition from manipulation strategies to cognitive strategies results in inefficient problem solving (Boulton-Lewis, 1998) with implications for mathematics achievement. Retrieval was included in the cognitive strategies category because cognitive strategies, such as decomposition and the standard algorithm, frequently involve retrieval of basic facts from memory as a part of the strategy. Given this and the evidence that retrieval loaded on a common factor with cognitive strategies, it made sense to include the few instances of retrieval in the cognitive strategy category. Aggregating to create categories for manipulative and cognitive strategy use was also advantageous in that it allowed us to use growth mixture modeling to examine this development.

We examined the development of attempted (both correct and incorrect) cognitive strategy use and correct cognitive strategy use because we expected gender to have a large influence on attempted cognitive strategy use. We expected that boys, who tend to be more competitive in mathematics classrooms (Seegers & Boekaerts, 1996), would attempt to use cognitive strategies, both successfully and unsuccessfully, more than girls would. We also expected, however, that fluency and accuracy would have a larger impact than gender on correct cognitive strategy use. We looked at attempted (both correct and incorrect) manipulative strategy use to determine whether different developmental trajectories emerged in comparison to cognitive strategy use and to determine the roles of gender, fluency, or accuracy on those developmental trajectories. Prior research indicated that girls would be more likely to use manipulatives, but poor fluency and accuracy could also explain the use of this strategy.

We did not examine correct manipulative strategy use because although we expected a decrease in manipulative strategy use as a result of the transition to cognitive strategies, we expected that the children would become more accurate over the 3-year period when they used these strategies. Given this, it was difficult to make predictions about correct manipulative strategy use or to interpret developmental patterns in correct manipulative strategy use.

The study addressed three questions about the development of manipulative and cognitive strategies and the impact of gender, fluency, and accuracy on strategy development:

1. How much variability is there in the developmental trajectories of strategies, and can growth mixture modeling identify distinct trajectories that apply to large subgroups of the population? In this study, we examined the development of strategies for complex arithmetic problems with the expectation that children would shift from strategies that use manipulatives to cognitive strategies over the period of the study. Given that prior research indicates that children progress from manipulative strategies to cognitive strategies at different rates, we expected to find multiple developmental trajectories of strategies for solving complex arithmetic problems.

2. Does gender influence the intercept and trajectory of arithmetic strategies and subsequent mathematics competency? In this study, we examined whether gender predicted the intercept of cognitive and manipulative strategies and the rate at which children adopted cognitive strategies and abandoned manipulative strategies. We also determined whether gender differences in the growth rate of strategies predicted achievement in boys and girls. On the basis of prior research, we expected a higher intercept and growth rate in cognitive strategy use, particularly for attempted cognitive strategy use, for boys and a higher intercept in manipulative strategy use for girls.

3. Do fluency and accuracy, as measured in the first year of the study, influence the intercept and trajectory of arithmetic strategy use and subsequent mathematics competency? Students who are more fluent and accurate on single-digit arithmetic problems were expected to show a higher growth rate in the acquisition of the more advanced cognitive strategies and higher achievement in fourth grade.
Method

Participants

Two hundred forty 2nd-grade students, 118 boys and 122 girls, participated in the first year of the study. Two hundred twenty-two children (109 boys and 113 girls, 93% of the original sample) participated in the second year of the longitudinal study. Two hundred six children (101 boys and 105 girls, 86% of the original sample) participated in the third year of the study. For this sample, missing data occurred when some children moved or were home schooled, but other children had incomplete data because they were not present on the day the data were collected. An examination of the data indicated no significant differences between the children with missing data and the children with no missing data. For the analyses, we used the Mplus (Version 6) program, which estimates missing data by default. A 20% attrition rate is average for studies that estimate missing data when doing growth mixture modeling (Bauer, 2007); therefore, we were well within acceptable limits for estimation of missing data.

Seventy-one percent of the sample was White, 24% was African American, 2% was Latino, and 3% was Asian. The mean age of the students at the beginning of the study was 7.5 years ($SD = 0.62$). Participating children were recruited with the written permission of their parents and the school. The schools were selected on the basis of their location within an hour drive from the university and with the goal of including minorities in the sample. Children of all ability levels and from all of the regular second grade classrooms in the participating schools were asked to participate, and all children who returned permission slips were included in the study. Of these children, only one child was dropped from the study because he could not complete the tasks. We did not collect data on disability status during the study. In all, children from seven schools (38 classrooms) from three counties in Georgia participated in the study. The percentage of economically disadvantaged students eligible for free or reduced lunch in the three counties was 51%, 57%, and 71%. Participating schools used an assortment of programs for mathematics instruction, with most schools using curriculums designed by Harcourt Press or Saxon Math.

Procedures and Measures

During the fall of each year, students’ arithmetic strategy use was assessed on double- and triple-digit arithmetic problems. For the first year of the study, response time (fluency) and accuracy when solving 10 single-digit arithmetic problems were also assessed. For these assessments, the children were interviewed individually outside the classroom. Most interviews, which were done by graduate students and Martha Carr, were completed within an hour. In April of the fourth grade school year, the children took the mathematics competency test required of all children in the state of Georgia. This information was collected from student records.

Strategy use. All interviews to assess children’s strategy use were videotaped for later coding. For each child in the second and third grades, the investigator presented and read two sets of problems, both printed on cards. The problems were presented randomly within each set. The first set consisted of 10 double- or triple-digit computation problems with solutions ranging from 3 to 595. Next, the investigator presented and read 10 (five addition, five subtraction) word problems with solutions ranging from 9 to 101. The problems were selected for a range of difficulty, on the basis of information from teachers about the grade appropriateness of the problems, with half of the problems in each set requiring children to carry or borrow. For the children in fourth grade, some of the easier problems used in the prior grades were dropped from the computation and word problems and new, more difficult problems were included. For the children in fourth grade, the computation problems consisted of 10 double- or triple-digit computation problems with solutions ranging from 59 to 1,075. In addition, 12 word problems were presented instead of 10, with three of the word problems requiring multi-step solutions. Pencil, paper, and plastic manipulatives (individual counters) were available for use. The children worked at their own pace to solve each problem using any strategy they wished to use. If a child was unable to solve the problem within 2 min, the investigator gave the child the option of moving on to the next problem. After a child solved a problem, the investigator asked the child how he or she solved it. The children did not receive feedback about the accuracy of their responses. The math problems are presented in the Appendix.

We created three strategy categories by determining the number of problems that students attempted (successfully or unsuccessfully) to answer using cognitive strategies, the number of problems students attempted (successfully or unsuccessfully) to answer using manipulative strategies, and the number of problems the students answered correctly using cognitive strategies. For all problems, strategies were categorized on the basis of a combination of children’s reports of strategy use, observation of children’s behavior, and observations of solution times. Although some problems can occur in retrospective reporting when students provide incomplete information about strategy use, this technique of observing strategy use and then asking children for a report of the strategy immediately after solution has been found to be a valid indicator of the strategies children use (Cooney & Ladd, 1992). When there were inconsistencies that the rater could not resolve, a second rater was consulted. If the student used more than one strategy, the strategy that resulted in the answer was coded.

Manipulative strategies. The manipulative strategy category comprised strategies for which children successfully or unsuccessfully used manipulatives (e.g., counting on with fingers, counting with counters) to get the answer. The use of paper and pencil resulted in a categorization of manipulative strategy use when the children created hatch marks to count or when the children wrote out the problem while using the standard algorithm and calculated answers for each column by counting on fingers or counters.

Cognitive strategies. The cognitive strategies categories comprised strategies involving children counting mentally or retrieving from memory (e.g., counting on mentally, decomposition) and for which there was no evidence that the child counted on

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1 We use the term response time instead of fluency in the Method and Results sections because fluency was measured as the amount of time needed to answer single-digit arithmetic problems. The use of response time makes it easier to understand the negative relationship between response time and most other measures. We use the term fluency in the introduction and Discussion section because that is the term usually used in the literature.
fingers or counters or used hatch marks on paper. We also looked for evidence of mental counting by observing students’ lip movements during solution, which would support a categorization of cognitive strategy use. If the children wrote down the problem as a part of the standard algorithm and counted mentally or retrieved the answer for each column, the strategy was categorized as cognitive. The correct cognitive strategy use category included the number of times students used cognitive strategies successfully. The attempted cognitive strategy use category included the number of times students used cognitive strategies either successfully or unsuccessfully.

Some children were unable to generate a strategy and were unable to give an answer to some problems. When this occurred, no strategy was coded. To assure interrater reliability, Martha Carr trained two independent raters to 95% agreement on data from 10 children. The two raters, both graduate students, then coded children’s strategies from the videotaped data. Interrater reliability between the two coders was established by having 25% of the videotaped data coded by both raters. Kappas for interrater reliability for the second-, third-, and fourth–grade data were .95, .96, and .90, respectively.

For each category, a score was created by summing the total number of times a student either attempted or correctly used the cognitive strategy or attempted to use manipulative strategies. In the second and third grades, scores for each of the three strategy categories could range from 0 to 20. In the fourth grade, the scores could range from 0 to 22. The scores for the fourth grade were rescaled (multiplied by 2,022) so that they could be compared with scores for the second and third grades.

Response time (fluency) and accuracy on single-digit arithmetic problems. After completing the strategy use interview, we presented students with 10 (five addition, five subtraction) single-digit problems to assess students’ response time (fluency) and accuracy in solving simple arithmetic problems. The problems, selected from grade-appropriate materials and approved by second-grade teachers, were representative of the single-digit problems familiar to these second graders. The problems (see Appendix) were presented on printed cards to the children and were read to the child. The children were told that they should solve these problems as quickly as possible and that they should not use their fingers. The counters, pencil, and paper were removed for this task. Response time was measured in seconds. We calculated response time (fluency) by averaging the response times for the correct problems. Low scores on the response time (fluency) measure indicate high fluency. We calculated accuracy by averaging the number of problems the children answered correctly. The fluency measure and the accuracy measure were not correlated in the second grade ($r = .08$).

Mathematics competency. To assess mathematics competency, we obtained the children’s scores on the mathematics portion of the Criterion Referenced Competency Test (CRCT) that was administered at the end of the fourth grade (Georgia Department of Education, 2010). The CRCT mathematics test comprised two sections, each with 35 problems. Testing took place in whole classes. The proctor read the questions and answers for the CRCT mathematics test; the pace of the test was determined by the proctor’s reading of the test items, with each item being read only once. The test was in multiple-choice format in which children must select from one of three answers. The total test time was about 120 min for both sections, with additional time for a break between sections.

The test is designed to assess how well students had acquired knowledge and skills set forth by Georgia’s performance standards, including a broad range of skills and knowledge related to computation and estimation, geometry, patterns/relations and algebra, problem solving, and number sense. Scores for the total test were used to assess the impact of predictor variables on a broad range of mathematics skills and knowledge. Following state scoring guidelines, we scored the children as not meeting the standard, meeting the standard, or exceeding the standard. Cronbach’s alpha for the CRCT measure as reported by the state was .92.

Results

Conventional growth modeling assumes that a single model accounts for all variations in the individual trajectories. Growth mixture modeling (GMM), in contrast, assumes that there are several subgroups in the population, and it explores qualitative differences in growth trajectories (e.g., Muthén, 2006; Muthén, Collins, & Sayer, 2001). To do this, GMM estimates latent groups or classes that have different developmental trajectories. In addition, GMM allows researchers to examine the impact of covariates on the intercept and slope of individual latent classes as well as the impact of class membership on a distal outcome. The technique is ideal for dealing with complex systems that might involve a number of variables that influence an outcome.

For the current study, in three GMM models, we examined whether there were latent classes as evidenced by different developmental trajectories of correct cognitive, attempted cognitive, and attempted manipulative strategy use. We were able to do this because although cognitive strategy use and manipulative strategy use were significantly correlated, the combination of manipulative and cognitive strategies did not equal the total number of problems. Some children did not give an answer to some problems, and children sometimes used procedures that were not viable strategies for solving the problem. In both cases, no strategy was recorded.

We included gender, response time (fluency), and accuracy as covariates to examine their impact on the initial status (intercept), rate of growth (slope), and latent class membership for the three strategy categories. In addition, we examined whether membership in particular classes predicted whether the children would meet standards, exceed standards, or not meet standards on the mathematics competency test (distal outcome).

Model Selection

There is still no consensus on the best method for determining the number of latent classes for GMM. Nylund, Asparouhov, and Muthén (2007) found that the Bayesian information criterion (BIC) and consistent Akaike information criterion (CAIC) performed the best among information indices for GMM, whereas Enders and Tofghi (2008) found that only sample-adjusted BIC worked reasonably well. Another issue intertwined with latent class enumeration is the way researchers treat the potential sources of heterogeneity, the covariates. One way is to use a two-step approach: first identifying latent class membership and subsequently regressing class membership on covariates. An alternative,
one-step approach is to include covariates in the model when classes are identified (Lubke & Muthén, 2007; Muthén, 2006). Both procedures have limitations. The two-step approach is less statistically sound; the one-step approach is difficult to use with very complex models. Because our model is not very complex, we used the one-step procedure for the current study.

Each model was assessed using the information-based criteria index, the reliability of classification into latent classes, the entropy index, and likelihood ratio test for the optimal number of classes, all of which are available in Mplus (Version 6; Muthén & Muthén, 1998–2010). The BIC and CAIC are used to compare model fit between nonnested models, with smaller values indicating a better model fit. The reliability of the classification was assessed by examining the average posterior probability of students being in a particular class. The entropy index shows the possibility of prediction of class membership given the observed variables. For the entropy index, values range from 0 to 1, and higher values indicate that the latent classes are more discriminative. The likelihood ratio test assesses the model and the optimal number of latent classes by comparing a $k$ versus $k - 1$ class model. We used the Lo–Mendell–Rubin test developed by Lo, Mendell, and Rubin (2001); a low $p$ value indicates that a $k - 1$ class model is rejected in favor of a $k$ class model. On the basis of these criteria, a three-class model was chosen for attempted cognitive strategy analysis, whereas two-class models were chosen for correct cognitive strategy and attempted manipulative strategy analyses (see Table 1).

The model for all three analyses is presented in Figure 1. For all three analyses, the model includes gender, response time (fluency), and accuracy, measured in the second grade, as covariates. It includes the distal outcome variable of mathematics competency as measured in the fourth grade as well as strategy use (correct cognitive, attempted cognitive, or attempted manipulative) measured in Grades 2 through 4. These variables were used, along with the covariates, to estimate the latent class membership, the initial status or intercept for strategy use, and rate of growth or slope. Latent class membership predicted performance on the mathematics competency test.

The mathematics competency test scores were converted into three categories (not meeting standards, meeting standards, and exceeding standards). This was done because this statistical technique requires a distal outcome to be a categorical variable. We applied the levels that the state uses in reporting students’ achievement.

To maintain consistency across the three models and improve interpretation, we labeled latent class membership so that lower numbers (e.g., membership in Latent Class 1) reflected less mature strategy use (less cognitive strategy use or more manipulative strategy use), whereas membership in higher classes (e.g., Latent Class 3) reflected more advanced strategy use (more cognitive strategy use or less manipulative strategy use).

### Estimated Means for Initial Strategy Use (Intercept), Rate of Growth (Slope), and Effect of the Covariates on Intercept and Slope

The estimated means for growth, initial strategy use, fluency, accuracy, gender, and latent class sizes are in Table 2. In all three analyses, the means of growth and initial strategy use were significantly different across latent classes ($p < .01$). The means of response time (fluency) were significantly different across latent classes ($p < .01$) in all analyses with the exception of between Latent Classes 1 and 2 in the analysis of the attempted cognitive strategy use model. The means of accuracy were significantly different across the two latent classes in the analysis of the correct cognitive strategy use model and between Latent Classes 1 and 3 as well as between Latent Classes 2 and 3 of the analysis of the attempted cognitive strategy use model. As can be seen in Table 2, accuracy and fluency were very similar for the two latent classes of attempted manipulative strategy use.

The estimated mean growth trajectories for correct cognitive strategy use, attempted cognitive strategy use, and attempted manipulative strategy use are presented in Figure 2. As can be seen in

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#### Table 1

Model Fit Indices for the Growth Mixture Modeling

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<th>No. of classes</th>
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<th>CAIC</th>
<th>Entropy</th>
<th>LMR p value</th>
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*Note.* BIC = Bayesian information criterion; CAIC = consistent Akaike information criterion; LMR = Lo–Mendell–Rubin test.
the figure, there were two latent classes for correct cognitive strategy use. Latent Class 1, with 70% of the students, began the second grade rarely using cognitive strategies correctly but substantially improved their correct cognitive strategy use over the 3-year period. Latent Class 2, comprising 30% of the sample, began the second grade correctly using cognitive strategies at a very high level and maintaining that high level throughout the 3-year period of the study.

For attempted cognitive strategy use, three latent classes were found (see Figure 2). Latent Class 3, making up 27% of the sample, showed consistently high levels of attempted cognitive strategy use over the 3-year period of the study. Latent Class 2, comprising 30% of the sample, showed consistently high levels of attempted cognitive strategy use. Latent Class 1, with 70% of the students, began the second grade attempting to use cognitive strategies at a higher rate. For each 1-s increase in response time (poorer fluency) increased their correct cognitive strategy use at a lower rate than children who had a higher response time (better fluency). For each 1-s increase in response time, the slopes for correct and attempted cognitive strategy use, children who had a higher response time (poorer fluency) increased their correct cognitive strategy use at a lower rate than children who had a lower response time (better fluency). For each 1-s increase in response time, the slopes for correct and attempted cognitive strategy use, children who had a higher response time (poorer fluency) increased their correct cognitive strategy use at a lower rate than children who had a lower response time (better fluency). For each 1-s increase in response time, the slopes for correct and attempted cognitive strategy use, children who had a higher response time (poorer fluency) increased their correct cognitive strategy use at a lower rate than children who had a lower response time (better fluency). For each 1-s increase in response time, the slopes for correct and attempted cognitive strategy use, children who had a higher response time (poorer fluency) increased their correct cognitive strategy use at a lower rate than children who had a lower response time (better fluency). For each 1-s increase in response time, the slopes for correct and attempted cognitive strategy use, children who had a higher response time (poorer fluency) increased their correct cognitive strategy use at a lower rate than children who had a lower response time (better fluency). For each 1-s increase in response time, the slopes for correct and attempted cognitive strategy use, children who had a higher response time (poorer fluency) increased their correct cognitive strategy use at a lower rate than children who had a lower response time (better fluency). For each 1-s increase in response time, the slopes for correct and attempted cognitive strategy use, children who had a higher response time (poorer fluency) increased their correct cognitive strategy use at a lower rate than children who had a lower response time (better fluency). For each 1-s increase in response time, the slopes for correct and attempted cognitive strategy use, children who had a higher response time (poorer fluency) increased their correct cognitive strategy use at a lower rate than children who had a lower response time (better fluency). For each 1-s increase in response time, the slopes for correct and attempted cognitive strategy use, children who had a higher response time (poorer fluency) increased their correct cognitive strategy use at a lower rate than children who had a lower response time (better fluency). For each 1-s increase in response time, the slopes for correct and attempted cognitive strategy use, children who had a higher response time (poorer fluency) increased their correct cognitive strategy use at a lower rate than children who had a lower response time (better fluency). For each 1-s increase in response time, the slopes for correct and attempted cognitive strategy use, children who had a higher response time (poorer fluency) increased their correct cognitive strategy use at a lower rate than children who had a lower response time (better fluency). For each 1-s increase in response time, the slopes for correct and attempted cognitive strategy use, children who had a higher response time (poorer fluency) increased their correct cognitive strategy use at a lower rate than children who had a lower response time (better fluency). For each 1-s increase in response time, the slopes for correct and attempted cognitive strategy use, children who had a higher response time (poorer fluency) increased their correct cognitive strategy use at a lower rate than children who had a lower response time (better fluency). For each 1-s increase in response time, the slopes for correct and attempted cognitive strategy use, children who had a higher response time (poorer fluency) increased their correct cognitive strategy use at a lower rate than children who had a lower response time (better fluency). For each 1-s increase in response time, the slopes for correct and attempted cognitive strategy use, children who had a higher response time (poorer fluency) increased their correct cognitive strategy use at a lower rate than children who had a lower response time (better fluency).
strategy use decreased 0.72 and 1.01, respectively, when gender and accuracy were held constant.

For the analysis of attempted manipulative strategy use, response time had a significant effect on the intercept (2.56, \( p < .01 \)) and slope (−1.28, \( p < .01 \)) of Latent Class 1. The effect on the intercept is clear: The higher the response time (poorer fluency), the more likely children were to attempt to use manipulative strategies in the second grade. In regard to the slope, for each second increase in response time, the slope is lower by 1.28, meaning that the children with the higher response time (poor fluency) had a higher rate of decrease in manipulative strategy use. This occurred because children with high response times in the second grade were using manipulatives at a very high rate at the beginning of the study (intercept), and this allowed them to abandon manipulative strategy use at the higher rate. They had more room to decrease manipulative strategy use within this class in comparison to children in this class who had lower response times and less manipulative strategy use.

The data on covariates indicates that response time (fluency) was the primary factor affecting rate of growth in use of cognitive strategies. Accuracy, in contrast, was a good predictor of initial use of cognitive strategies; however, it did not have a significant effect on the growth rate. Gender had an effect on the initial use of strategies, but only in latent classes in which students tended to use cognitive strategies less often than students in other latent classes.

Effect of Covariates on Class Membership

Gender did not significantly predict class membership in any of the analyses. Response time and accuracy had significant effects on latent class membership, although these effects varied depending on the class and the type of strategy examined. The probability of latent class membership as a function of response time (fluency) and accuracy for correct cognitive strategy use and attempted cognitive strategy use are shown in Figures 3 and 4, respectively. Figure 5 shows the probability of latent class membership as a function of fluency for attempted manipulative strategy use.

In the analysis of correct cognitive strategy use, both response time and accuracy had an effect on class membership (see Figure 3). The GMM indicated that for each unit increase in response time, the log odds of being in Latent Class 1 as opposed to Latent Class 2 would increase by 1.07. Likewise, a unit increase in accuracy (by 1%) would decrease the log odds of being in Latent Class 1 as opposed to Latent Class 2 by 0.04.

For the analysis of attempted cognitive strategy use (see Figure 4), response time and accuracy did not have significant effects on
membership in Latent Class 1 as compared with Latent Class 2. Children in these latent classes differed only in the means of their slopes and the intercepts. Response time (fluency) and accuracy, however, had an effect on whether an individual would be in Latent Class 1 as opposed to Latent Class 2 and whether an individual would have membership in Latent Class 2 as opposed to Latent Class 3. For example, a student’s log odds for being in Latent Class 1 as opposed to Latent Class 3 would be reduced by 0.06 for each 1% increase in accuracy and would be increased by 1.01 for each second that response time increased.

For the analysis of attempted manipulative strategy use, only the effect of response time (fluency) on latent class membership was significant. For each reduction in response time of 1 s, the log odds of being in Latent Class 1, as opposed to Latent Class 2, would be reduced by 0.47.

**Effect of Latent Class Membership on Fourth Grade Mathematics Competency Test**

As can be seen in Figure 6, being in the more advanced Latent Class 2 of correct cognitive strategy use and Latent Class 3 of attempted cognitive strategy use substantially increased the probability that a child would meet or exceed the standards set for the mathematics competency test. Most children, however, were in Latent Class 1. Members of Latent Class 1 of correct cognitive strategy use had a .26 probability of not meeting the standards for the mathematics competency test; similarly, members of Latent

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**Figure 4.** Probabilities of latent class membership as a function of fluency (top) and accuracy (bottom) for the attempted cognitive strategy use.

**Figure 5.** Probabilities of latent class membership as a function of fluency for the attempted manipulative strategy use.

**Figure 6.** Probabilities for mathematics competency test levels by latent class and analysis.
Class 1 of attempted cognitive strategy use had a .32 probability of not meeting the standards of the mathematics competency test. These data indicate that the probability of meeting the standards for the mathematics competency test in the fourth grade are substantially increased if the student begins the second grade attempting and correctly using cognitive strategies at the higher rate evident in the strategy use of children in Latent Classes 2 and 3.

For attempted manipulative strategy use, the number of children in Latent Class 1 was much smaller than the number of children in Latent Class 2, but children in Latent Class 1 had a .33 probability of not meeting the standards and .67 probability of meeting standards, whereas children in Latent Class 2 of manipulative strategy use had a .14 probability of not meeting standards, a .72 probability of meeting the standards for mathematics, and a .13 probability of exceeding the standards. Most children were in Latent Class 2 of manipulative strategy use and had a good chance of meeting mathematics competency standards as a function of membership in this class.

Discussion

To date, there has been limited research on strategies for complex arithmetic problems and no longitudinal studies examining the developmental trajectories of these strategies. Furthermore, although fluency, accuracy, and gender are linked to strategy use and mathematics achievement, no research has examined how they impact the development of arithmetic strategies. In the current study, we examined whether the transition from the use of manipulative strategies to cognitive strategies was homogeneous or heterogeneous; whether differences in the developmental trajectory of strategies predicted fourth-grade mathematics competency; and the impact of fluency, gender, and accuracy on the initial state and rate of growth of arithmetic strategies for complex problems. Our analyses indicated that there are multiple developmental trajectories for complex arithmetic strategies and that these developmental trajectories had significant implications for later mathematics competency. Specifically, membership in the largest latent classes (normal development) resulted in many children not meeting standards for mathematics achievement, and some children appear to be very slow in developing cognitive strategies. Our data indicate that developmental trajectories and differences in later competency were significantly influenced by fluency and accuracy as measured in the second grade, suggesting that early skills have long-term consequences for students.

For correct and attempted cognitive strategy use, we found that the patterns were heterogeneous in that children could start the second grade at very different levels of correct and attempted cognitive strategy use, only to have these differences lessen as they entered the fourth grade. Despite decreases in differences in strategy use, differences on the mathematics competency test remained, indicating that early differences in cognitive strategy use had long-term influences on mathematics competency.

Analyses of manipulative strategies showed a very different pattern of development, with the largest latent class showing a consistent drop in attempted manipulative strategy use and a second latent class showing a slight increase in attempted manipulative strategy use across the 3-year period of the study. The two latent classes of manipulative strategies resulted in different outcomes on the fourth-grade competency test with children who slightly increased their use of manipulatives over the period of the study having very poor outcomes. The failure to move away from manipulatives during this 3-year time period predicted a broad range of problems with mathematics learning as measured by the mathematics competency test.

Our data suggest that for most children, it is not the way they moved away from manipulatives but the way they acquired cognitive strategies that most influenced later mathematics competency. Most children were in the latent class of manipulative strategy use that resulted in a good probability of success on the fourth-grade mathematics competency test. In contrast, when looking at attempted and correct cognitive strategies, most children were in the latent classes that had the highest probability of not meeting the standards. Our data indicate that to have improved chances of success in mathematics, children need to begin second grade using cognitive strategies at a higher rate than most children currently do.

Gender, fluency, and accuracy played different but significant roles in the developmental trajectories of complex arithmetic strategies and mathematics competency. Fluency had a significant impact on the rate of growth of strategies. For analyses of all three strategy categories, being in the highest functioning latent class was highly dependent on fluency. Gender and accuracy, in contrast, predicted the initial status of the child in second grade but not the rate of growth. Both accuracy and fluency influenced mathematics competency in fourth grade through latent class membership. Success on the fourth-grade mathematics competency test, therefore, appears to be dependent on the fluency and accuracy for basic arithmetic facts that children develop by the beginning of the second grade.

Development of Strategies

The results of the analysis of attempted manipulative strategy use indicate that most children, as would be expected, abandon the use of manipulatives to solve arithmetic problems over the 3-year period. It was surprising to find a latent class of children who maintained and somewhat increased their attempted use of manipulatives. For the most part, this was a particularly poorly performing latent class of children who were much more likely than children in other classes to fail the mathematics competency test. Surprisingly, the accuracy and fluency (response time) of the children in the two classes of manipulative strategy use was not different in the second grade. Fluency (response time), however, was found to predict whether children would be in Latent Class 1 or Latent Class 2 of manipulative strategies. These findings extend prior research indicating a link between fluency and early mathematics performance (e.g., Baroody, 2006; Brownell, 1935; Gersten & Chard, 1999; Jordan, 2007) by suggesting that it is the impact of fluency on the developmental trajectory that influences outcomes. The results are in line with prior research (e.g., Geary et al., 2000) indicating the need to consider more than a single assessment of strategy use when predicting later performance. In this case, average fluency as assessed in the second grade would not have been sufficient to predict later manipulative strategy use.

In the analyses of the attempted and correct cognitive strategies, there was a latent class of children that included most of the children in the sample. Children in these latent classes tended to start the second grade rarely attempting or correctly using cogni-
tive strategies. Although the children in Latent Class 1 caught up with the children in Latent Class 2 by the time they entered the fourth grade, membership in Latent Class 1 did not result in a positive outcome for a large percentage of these children. Twenty-six percent and 31% of the children in Latent Class 1 of correct and attempted cognitive strategies, respectively, failed the mathematics competency test. Thus, our findings indicated that what appears to be the normal pattern of development does not serve many children well.

The model of attempted cognitive strategy use included a smaller class of children who differed from the highest performing group in terms of the initial level of attempted cognitive strategy use and rate of change in strategy use over the 3-year period. They began the second grade attempting to use cognitive strategies at higher rates than children in the largest latent class (Class 1) but were not performing at the level of the high performers in Latent Class 3. The advantage they showed at the beginning of the second grade on attempted cognitive strategies served them well in that it resulted in a higher percentage of these children meeting mathematics standards even though their attempted cognitive strategy use was almost identical to that of the children in Latent Class 1 in the fourth grade. The existence of this latent class suggests that simply trying to use the more advanced cognitive strategies appears to produce an advantage in later years.

In our study, one of the groups was made up of children who were attempting and correctly using cognitive strategies at a high rate at the beginning of the study. This finding is in line with previous work on individual differences in strategy use indicating that higher performing children acquire and use more mature strategies at an earlier age (Geary & Brown, 1991; Zimmerman & Martinez-Pons, 1990). Our data indicated that this group tended to show less quantitative change over time as assessed by the rate of growth of correct or attempted cognitive strategies in comparison to children in other classes. The problems presented to children in this study were not sufficiently challenging to this latent class of children. The inclusion of more challenging problems might have produced evidence of growth over time.

Role of Fluency

Our data are in line with prior research indicating the importance of fluent retrieval and computation in mathematics ability (Bull & Johnston, 1997; Jordan et al., 2003) and support efforts to improve fluency in children (Fuchs et al., 2005). Our data suggest that fluency plays a significant role in the development of more advanced mathematics strategies and subsequent mathematics competency, perhaps as a result of the better organization in memory of basic arithmetic facts.

Fluency (response time) had a significant impact on the rate at which children increased their attempted and correct use of cognitive strategies. It also influenced class membership for these strategy categories. In regard to its impact on the rate of growth, fluency predicted the rate of growth for children in the largest latent class (Latent Class 1) that began the second grade rarely attempting or correctly using cognitive strategies but showing considerable growth in these strategies across the 3-year period. It also predicted the latent class, and therefore the developmental trajectory, of attempted and correct cognitive strategies. Taken together, this shows that early fluency on basic arithmetic facts has long-term consequences for the development of more advanced strategies and for later mathematics competency.

Fluency did not have an impact on the rate of growth of attempted and correct cognitive strategies for children in higher performing latent classes, perhaps because these children began the study already using these strategies at a relatively high rate. The lack of relationship between fluency and performance in higher latent classes, however, should not be interpreted to mean that it is not an important contributor to performance for these children. These children had high fluency and accuracy to begin with and were significantly higher in comparison to the children in Latent Class 1. Fluency and accuracy may have been more important predictors in Latent Classes 2 and 3 if we had assessed these children on more advanced mathematics, such as division or fractions. Future research needs to determine the relationship between fluency and performance in higher performing groups of children.

Fluency predicted the initial state and rate of growth of children in Latent Class 1 of attempted manipulative strategy use. The worse the fluency (high response time), the more likely children were to attempt to use manipulative strategies in the second grade. In regard to rate of growth, children who had poorer fluency (higher response times) abandoned manipulative strategies at a higher rate than more fluent children over the 3-year period of the study. This is because children within this class who had poor fluency in second grade were using manipulatives at a very high rate at the beginning of the study and, therefore, had more opportunity to decrease manipulative strategy use.

Role of Accuracy

The failure of accuracy to predict rate of growth in the analyses should not be interpreted to mean that accuracy on simple arithmetic problems is not important for strategy development and mathematics competency. Accuracy determined both the initial state of strategy use and, through that, class membership, with higher accuracy being indicative of higher latent classes. It may not have predicted rate of growth because children could be quite accurate when counting on fingers or counters, but this form of accuracy did not predict growth in cognitive strategy use. Thus, children’s accuracy at the beginning of the second grade significantly predicted their performance 2 years later.

Accuracy predicted whether children would be in the highest functioning classes. In regard to its impact on attempted cognitive strategy use, accuracy predicted whether children would be in the highest functioning Latent Class 3 as opposed to Latent Classes 1 and 2. It predicted class membership for correct cognitive strategy use with children, with high accuracy being more likely to be in Latent Class 2. Children who were more accurate were more likely to show a pattern of very high attempted and correct cognitive strategy and to be more successful on the fourth-grade assessment. Specifically, to be in the highest performing class in terms of strategy use and competency, children needed to begin the second grade with near perfect accuracy.

Role of Gender

Boys were more likely to attempt and correctly use cognitive strategies at the beginning of the study. These results are in line with previous research indicating gender differences in strategy
use, with boys using more cognitive strategies and retrieval (Carr, Jessup, & Fuller, 1999; Fennema et al., 1998). The same prior research had indicated gender differences for the use of manipulative strategies, but that was not evident in this sample, with girls beginning the study attempting to use manipulatives at the same rate as boys. The lack of gender differences for manipulative strategy use may be the result of both boys and girls finding the complex problems to be challenging, causing them to revert to more familiar manipulative strategies. Research by Carr and Davis (2001) found that this occurred when first graders worked on more difficult problems.

Gender did not predict class membership so that girls were almost equally represented in all latent classes. Prior research had indicated that kindergarten boys were more likely than girls to be in higher performing classes for number sense and nonverbal calculation (Jordan et al., 2006) and that older males tend to dominate in the highest performing levels of mathematics (Bennet, 1992). Our results do not indicate that boys dominate the most advanced latent classes of strategy use (Latent Class 2 for correct cognitive strategy use and Latent Class 3 for attempted cognitive strategy use). In line with the work of Royer et al. (1999), our data indicate that for correct and attempted cognitive strategy use, it was the speed with which the children were able to answer simple arithmetic problems (fluency) as well as accuracy that was critical. In terms of attempted manipulative strategy use, it was accuracy and not gender that was the big predictor of latent class membership.

How is it that gender did not predict latent class membership for attempted and correct cognitive strategy use when it predicted initial cognitive strategy use in the second grade? This outcome would occur when girls attempt and correctly use cognitive strategies less often than boys within each latent class in the second grade. Although girls used cognitive strategies less often than boys did, the rate of growth for cognitive strategy use was influenced by response time (fluency) and not gender. Thus, gender differences that may be evident at a given point in time may not have implications for later development.

Gender did not predict the slope for attempted and correct cognitive strategy use, but it did predict the rate of growth for attempted manipulative strategy use for Latent Class 1. In regard to attempted manipulative strategy use, girls in Latent Class 1 abandoned the use of manipulative strategies more slowly than boys did over the period of the study. It is not clear why girls were slower to abandon manipulative strategy use in this class. They have been found to be less fluent than boys (Carr, Steiner, Kyser, & Biddlecomb, 2008; Royer et al., 1999), so they may have continued to use manipulatives to assure accuracy. Our data suggest that accuracy and fluency, as opposed to gender, determine differences in strategy use.

Limitations

This sample included a latent class of children who were, by the beginning of the second grade, performing at a sufficiently high level in terms of strategy use, fluency, and accuracy that we could not assess substantial change over the 3-year period. A better understanding of the development of strategies for this latent class of students awaits a study with more difficult tasks, perhaps including division and multiplication. We suspect that fluency and accuracy would be found to predict improvement in performance in this group but do not have the evidence here to test that assumption.

The current study only involved three data points, making it impossible to assess the impact of fluency and accuracy as it changed over the 3-year period on the emergence of cognitive strategies. A fourth time point would have allowed us to use more advanced models and time-variant covariates and to examine quadratic growth. Prior longitudinal research indicates that as average children transition from counting strategies to retrieval strategies, the speed with which they count and retrieve answers also increases (Geary, Brown, & Samaranayaka, 1991). A fourth time point for data collection would allow us to determine how increases in fluency and accuracy predict the transition to more advanced strategies and whether the transition influences fluency and accuracy. Likewise, data collection at four time points would allow us to examine how changes in manipulative strategies helped or hindered the emergence of cognitive strategies. When this study was initially designed, however, GMM was not initially planned as the method of analysis.

Our study considered only quantitative growth. We looked only at the number of strategies used by the children and collapsed across a range of strategies into manipulative and cognitive strategies. Not all strategies are the same. Some strategies, such as decomposition, reflect a deeper conceptual understanding of number (e.g., Steffe et al., 1988), whereas others (e.g., counting all) reflect a very concrete representation of number. A closer look at specific strategies would be helpful in understanding why some children used cognitive strategies and were successful on the mathematics competency test while others were not. The size of the sample, unfortunately, did not allow us to examine individual strategies in such a way.

A number of other variables, such as conceptual knowledge, working memory, spatial ability, and instruction, also have been found to influence the development of arithmetic strategies. Although the current study provided evidence that fluency and accuracy were important for the development of arithmetic strategies, future research is needed to examine how other variables may compare with fluency and accuracy as predictors of strategy development and mathematics achievement.

This study is correlational in nature. Although it is clear that fluency and accuracy predict the development of strategies and mathematics achievement, we must take care in concluding that instruction to improve fluency and accuracy will improve the development of mathematics strategies and mathematics achievement. Experiments involving interventions designed to improve fluency and accuracy are needed to determine whether and how improvements in fluency and accuracy affect strategy use and achievement.

Conclusion

The latent classes comprised children from different classrooms and schools, so school characteristics did not determine class membership. Nevertheless, we should be able to influence the development of arithmetic strategies through instruction. The data point to the importance of fluency and accuracy for the development of arithmetic strategies for complex problems during the early elementary school years. Our data support the call (e.g.,
Kilpatrick, Swafford, & Findell, 2001; National Council of Teachers of Mathematics, 2000, 2006; National Mathematics Advisory Panel, 2008) for teachers to focus on improving fluency and accuracy during the first and second grades. It also supports the existence of different learning trajectories (Clements & Sarama, 2004) that are a function of individual differences in instruction and child characteristics. We need to know more about the mal- leability of these differences to develop instructional activities that will improve early arithmetic ability.

Taken together, our findings indicate that individual differences in strategy use, fluency, and accuracy at the beginning of the second grade influence performance through the end of the fourth grade. Furthermore, poor accuracy and fluency in second grade is a harbinger of slowed development of strategies for the complex arithmetic problems children encounter in second, third, and fourth grades and an increased likelihood of poor performance in later elementary school mathematics.

References


Appendix

Items Used for the Computation, Word Problem, and Fluency Assessments

Computation Problems for Second and Third Grades

\[
\begin{align*}
41 + 13 & \quad 85 - 14 \\
82 + 7 & \quad 34 - 21 \\
89 + 99 & \quad 703 - 108 \\
306 + 188 & \quad 20 - 17 \\
97 + 3 & \quad 36 - 5
\end{align*}
\]

Word Problems for Second and Third Grades

1. Farmer Jones has 38 chickens and 10 ducks. How many birds does he have altogether?
2. Miss Smith’s second-grade class has 18 students. Miss Sawyer’s second-grade class has 19 students. How many students are there in the second grade?
3. John ate 87 French fries from his plate and 14 from his brother’s plate. How many French fries did John eat?
4. Chris has 16 kids on his soccer team. Cheryl has 20 kids on her soccer team. If they played a game together, how many kids would be playing soccer?
5. The computer lab at school has 23 computers. If 13 more computers are added, how many computers will be in the lab?
6. Brandon has 96 marbles. He plays with them outside and loses 15. How many does he have left?
7. Will’s father buys a package of 25 carrot sticks at the store. Will eats 6. How many carrot sticks are left over for Will’s sister?
8. Martha and Brian are making necklaces in art class. Brian has 45 beads and gives Martha 27. How many beads does Brian have left to make his necklace?
9. The school lunchroom buys 72 apples for lunch. They serve 63 of them at lunch. How many apples are left over for tomorrow’s lunch?
10. The pet store has 55 parakeets in a cage. Twelve of them fly away. How many are left?

Computation Problems for Fourth Grade

\[
\begin{align*}
89 + 99 & \quad 81 - 22 \\
59 + 29 & \quad 174 - 69 \\
111 + 79 & \quad 384 - 286 \\
450 + 625 & \quad 85 - 14 \\
306 + 188 & \quad 703 - 108
\end{align*}
\]

Word Problems for Fourth Grade

1. John ate 87 French fries from his plate and 14 from his brother’s plate. How many French fries did John eat?
2. The computer lab at school has 23 computers. If 13 more computers are added, how many computers will be in the lab?
3. On Monday Ben made 43 sandwiches. Ray made 12 more sandwiches than Ben. Rachel made 24 more than Ray. How many sandwiches did Rachel make?
4. John scores 651 points on a video game. Lori scores 418 points more than John. Alex scores 163 more points than Lori. How many points does Alex score?
5. The distance between Georgetown and Lincoln Park is 37 miles. The distance between Lincoln Park and Athens is 79 miles. What is the distance between Georgetown and Athens?
6. Sandy had 59 stickers. Her father gave her 28 more stickers for her birthday. How many stickers did Sandy have then?
7. Raymond’s lunch break at school is 55 minutes long. He spent 17 minutes in the hot lunch line and 19 minutes eating lunch. How much time did he have left?
8. Jesse writes two pages on a computer. He writes 234 words on page 1 and 188 words on page 2. How many more words does he need for a 500-word story?
9. Martha and Brian are making necklaces in art class. Brian has 45 beads and gives Martha 27. How many beads does Brian have left to make his necklace?
10. The school lunchroom buys 72 apples for lunch. They serve 63 of them at lunch. How many apples are left over for tomorrow’s lunch?
11. The Art Club raised $437 by selling cookies. The Music Club raised $673 by washing cars. How much more did the Music Club raise?
12. The school has 436 pencils to give away and they give away 194 pencils. How many pencils do they have left?

Problems for Fluency and Accuracy

\[
\begin{align*}
5 + 1 & \quad 7 - 0 \\
2 + 3 & \quad 6 - 2 \\
8 + 2 & \quad 9 - 8 \\
7 + 4 & \quad 8 - 5 \\
0 + 9 & \quad 4 - 3
\end{align*}
\]

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